

# RiaSoR2

RELIABILITY IN A SEA OF RISK

## Reliability Evaluation of CorPower Pre-tension Cylinder using VMEA

December 2018



CorPower C3 WEC deployment at EMEC Scapa Flow site © Colin Keldie

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## Executive Summary

The overall objective of the RiaSoR 2 work package 4 (Data processing) is to provide methodologies for reliability evaluation and data processing to the Monitoring Framework (WP2). This report treats reliability methodologies and presents a case study that evaluates two design criteria for the structural reliability of the double rod pre-tension cylinder in CorPower Ocean's C3 half-scale prototype Wave Energy Converter (WEC). The pre-tension cylinder is subjected to internal pressure and the required wall thickness is assessed for static strength and fatigue strength design cases. The design calculations at CorPower are based on the pressure vessel standards. The case study re-evaluates the reliability assessment using Variation Mode and Effect Analysis (VMEA). The safety requirements based on the standard and on the VMEA assessment are evaluated, compared and discussed. The purpose of the re-evaluation is firstly to compare the VMEA methodology to the reliability design according to the pressure vessel standard regarding the safety judgements, and secondly to investigate if the VMEA evaluation gives a better ground for improvements and updating, compared to the standard. Additionally, the results will feed into the continued work on condition monitoring and maintenance planning.

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# 1 Introduction

## 1.1 RiaSoR background

The goal of the RiaSoR project is to consistently learn from the physical interactions between the devices and their environments, while embedding this understanding and building robustness into marine energy technology designs to improve reliability.

Marine energy devices operate in harsh environments but still need to perform reliably and produce an expected amount of energy, which gives rise to huge engineering challenges.

The OceanERANET-funded RiaSoR 2 project will use the theoretical reliability assessment guideline for wave and tidal energy converters (WEC/TEC) developed in RiaSoR1 and apply it to the field.

This will enable WEC/TEC developers to validate their findings, and establish a practical condition based monitoring platform to prepare for future arrays where big data handling and processing will be vital to drive down operational expenditures (OPEX).

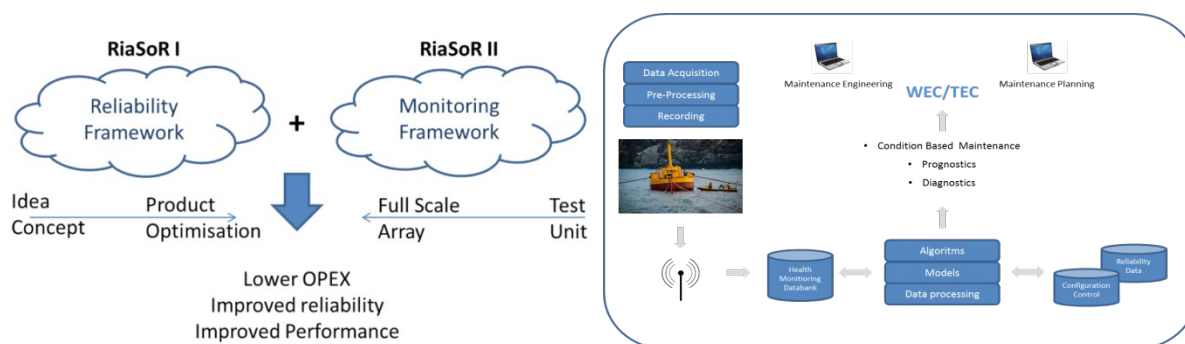


Figure 1: RiaSoR 1 & RiaSoR 2 overview.

The RiaSoR 1 reliability guideline built upon established practices from the automotive industry where a monitoring framework is applied to a fleet of test-vehicles. Through design iterations, the reliability is improved and a final reduced set of sensors are deployed in the commercial vehicle.

For RiaSoR 2, the chosen components for monitoring are equipped with several sensors to collect the required data, which will then be fed into the reliability process to reduce uncertainties. Sea tests act as case studies to feed the methodologies and training into the guideline. The findings from this will then be trialled with the other developers.

The key objective of the RiaSoR 2 project is to offer a comprehensive suite of testing methodologies to wave and tidal developers that will enable a systematic approach to achieve optimal reliability and performance, while minimising cost and time-to-market.

## 1.2 Work package 4: Data processing – aim and scope

The objectives of WP4 (Data processing) is to provide methodologies for reliability evaluation and data processing to the Monitoring Framework (WP2). The data from monitoring systems and

simulation tools will be processed in a unified way to get useful and reliable data for the identified critical components. The objective is to provide input to the design iterations during test phases as well as to update the reliability assessment in a commercial deployment (array configuration). WP4 provides methodologies as well as performs reliability assessments on the pilots.

### 1.2.1 Overall description and implementation methodology

WP4 will support the development of the data post-processing tools, processing large amounts of data covering loads and system dynamics from on-shore testing in HIL-rig and ocean testing to ensuring that the interpretation of both the outputs of the numerical tool and the field data analysis are well aligned with industry best practices. Based on the data, the VMEA reliability assessment from RiaSoR 1 will be validated and updated.

WP4 is the primary receiver of the reliability framework developed in RiaSoR I, and interacts with other WPs:

- WP2 (Monitoring Framework) – defining methodologies for processing data for use in reliability calculations and feed them into WP2 (Monitoring Framework);
- WP3 (Numerical Tool for Load Assessment) – using the data output for reliability updates and make sure optimal mutual benefits of WP3 and WP5 are reached;
- WP5 (Monitoring System) – retrieve necessary data to perform reliability update, make sure WP3 and WP5 are aligned when it comes to simulations and measurements.

## 1.3 Deliverable description

The aim of this case study is to evaluate the structural reliability of the double rod pre-tension cylinder in CorPower C3 half-scale prototype Wave Energy Converter (WEC) using Variation Mode and Effect Analysis (VMEA). The design calculations are based on the pressure vessel standards. The safety requirements based on the standard and on the VMEA evaluation are evaluated, compared and discussed. The VMEA methodology applied is based on the *Reliability Guidance for Marine Energy Converters*, (Johannesson, 2016).

### 1.3.1 Presumptive failure causes

Several presumptive failure causes of the pre-tension cylinder have been identified and investigated by CorPower. The investigations were made with respect to the demand on the half-scale prototype of survival for one year in service for the testing programme. The evaluations were based on criteria from pressure vessel standards including predetermined safety factors.

The report will re-evaluate two of the results by means of the probabilistic VMEA tool, namely

1. **Calculation of the required wall thickness of the cylinder barrel subjected to internal pressure acc. to EN 13445-3:2014, Sec. 7.**
2. **Endurance limit calculation acc. to EN 13445-3:2014, Sect. 17. Calculation of the wall thickness of the pre-tension cylinder subjected to internal pressure.**

The purpose of this re-evaluation is firstly to compare the VMEA methodology to the established reliability design tools regarding the safety judgements, and secondly to investigate if the VMEA

evaluation gives a better ground for improvements and updating, compared to the standard. Additionally, the VMEA will feed into the condition monitoring and maintenance work.

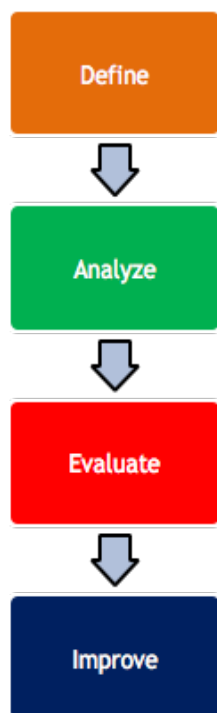
## 1.4 VMEA methodology

VMEA is a probabilistic method that studies the variation and uncertainty around a nominal design. Based on all variation and uncertainty sources, the methodology determines a statistical safety distance that, together with additional engineering risk judgements, gives a proper safety factor against eventual failure. The statistical safety distance is constructed by means of a confidence interval, in turn determined from an overall standard deviation for a suitable critical load/strength-function.

The VMEA approach represents a first order, second moment reliability method. “First order” is due to the fact that the influence of each term is approximated by one single linear term, and “second moment” is that the probabilistic influence is approximated by second moment statistics, variances and covariances. The VMEA method was first presented in (Chakhunashvili et al., 2004; Johansson et al., 2006) and further developed in (Chakhunashvili et al., 2009; Johannesson et al., 2009; Svensson et al., 2009). An adaptation to marine energy applications is found in (Johannesson, 2016), while general presentations of the methodology are given in (Bergman et al., 2009; Johannesson et al., 2013; Johannesson & Speckert, 2013; Svensson & Johannesson, 2013).

### 1.4.1 VMEA work process – 7 steps

The work process can be grouped into four activities “Define-Analyse-Evaluate-Improve”, as illustrated below. According to the *Reliability Guidance for Marine Energy Converters*, (Johannesson, 2016), we follow the seven steps for the VMEA evaluation.



1. **Target Function Definition.** For example, life of a component, maximum stress or largest defect.
2. **Uncertainty Sources Identification.** Identify all sources of uncertainty (scatter, statistical, model).
3. **Sensitivity Assessment.** Evaluate the sensitivity coefficients of the sources of uncertainty.
4. **Uncertainty Size Assessment.** Quantify the size of the different sources of uncertainty.
5. **Total Uncertainty Calculation.** Combine the contributions from all uncertainty sources.
6. **Reliability and Robustness Evaluation.** Find the dominating uncertainties or derive safety factors.
7. **Improvement Actions.** Identify uncertainty sources that are candidates for improvement actions.

Although the core VMEA methodology is steps 2-5, problem definition (step 1), reliability evaluation (step 6) and improvement work (step 7) are equally essential in the design process. Therefore, all seven steps are included in the overall VMEA methodology to cover the design and improvement cycle illustrated in Figure 2.

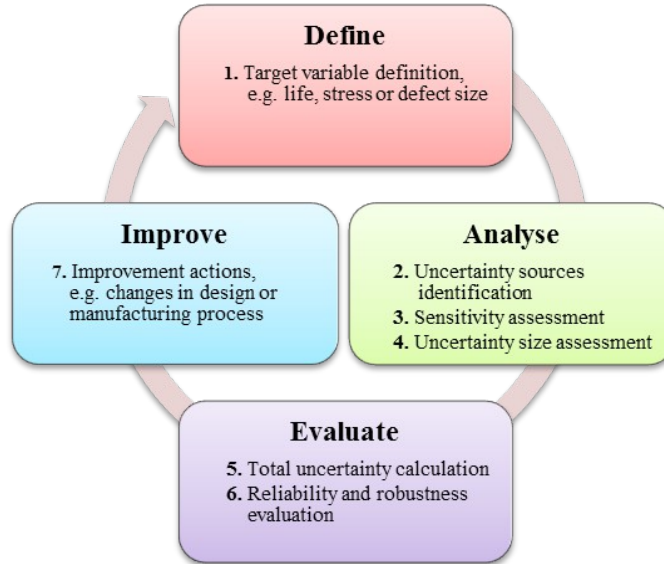


Figure 2: VMEA in the design and improvement cycle.

### 1.4.2 Mathematical principles of VMEA

The method is based on characterising each source by a statistical standard deviation and calculating its sensitivity with respect to the target variable, e.g. fatigue life or maximum stress. From the target variable the standard deviation, being the square root of the variance, is found using the *Gauss' approximation formula*

$$\text{Var}[\text{'target variable'}] \approx c_1^2 \text{Var}[x_1] + c_2^2 \text{Var}[x_2] + \dots + \text{Covariances} \quad (1)$$

This formula gives the variance of the target function  $f$  as the sum of variance contributions from different influencing variables  $x_i$ , each described by its own variance together with its influence of the function in question, by means of its sensitivity coefficient  $c_i$ . Also covariances between the influencing variables contribute, which however can usually be neglected or avoided by formulation.

In summary, the VMEA method combines input uncertainties into the total prediction uncertainty, denoted  $\tau$ , which is obtained by the root sum of squares (RSS) of the uncertainties (neglecting the covariances)

$$\tau = \sqrt{\tau_1^2 + \tau_2^2 + \tau_3^2 + \dots} = \sqrt{c_1^2 \sigma_1^2 + c_2^2 \sigma_2^2 + c_3^2 \sigma_3^2 + \dots} \quad (2)$$

where  $\tau_i$  is the resulting uncertainty from source  $i$  and is calculated as the product of the sensitivity coefficient  $c_i$  and the uncertainty  $\sigma_i$  of source  $i$ . Note that VMEA is a so-called second-moment method since it uses only the standard deviation to characterise the distribution of the uncertainty sources.

## 1.5 VMEA in product development

The probabilistic basis for the methodology, the Gauss approximation formula given above, may be simplified in an initial design stage where standard deviations and sensitivity coefficients are difficult to assess. The VMEA method is evolving through three different phases as shown in Figure 3, namely 1) basic VMEA, in the early design stage when little is known about variations, 2) enhanced VMEA, further in the design process when the sources of variation can be better identified, and 3) probabilistic VMEA, in the later design stage when detailed information is available for variations.

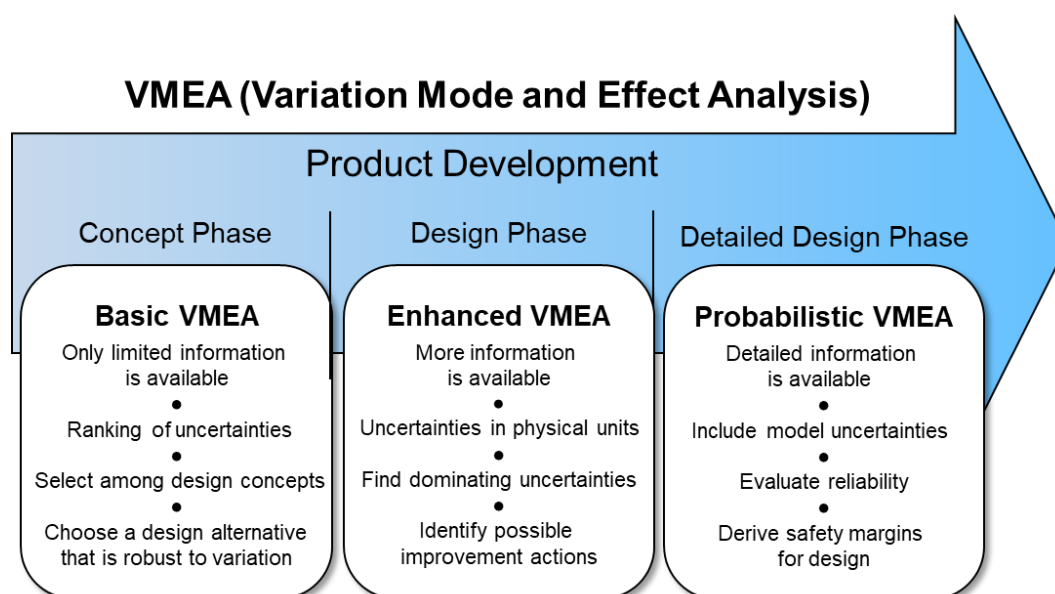


Figure 3: VMEA in different design phases.

### 1.5.1 Basic VMEA

The simplest approximation is called the Basic VMEA where standard deviations and sensitivity coefficients are replaced by scores, i.e. relative numerical engineering judgements about uncertainty and sensitivity, respectively. The Basic VMEA can be built up from a cooperative brain storm session. It gives a qualitative picture of uncertainty distribution between different sources and be used for prioritisation for further studies.

### 1.5.2 Enhanced VMEA

A refinement of the Basic VMEA may be done by quantifying uncertainties by judging their standard deviations by means of standard rules and judge sensitivities by fundamental physical knowledge. This analysis is called an Enhanced VMEA and can be used for a preliminary assessment of a safety factor needed for taking the studied uncertainties into account.

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### 1.5.3 Probabilistic VMEA

A further refinement, called the Probabilistic VMEA, is developed by getting more information about the most critical uncertainty sources. Standard deviations are obtained by more detailed studies of empirical results. Sensitivity coefficients are found from numerical sensitivity studies or differentiation of physical/mathematical models. The result of such an analysis give an estimate of the resulting total uncertainty and a corresponding statistical safety factor.

## 2 Static strength case

We will here re-evaluate the first case which represents static strength design:

1. Calculation of the required wall thickness of the cylinder barrel subjected to internal pressure acc. to EN 13445-3:2014, Sec. 7.

### 2.1 Target function definition and nominal values

The reliability in this case is regarded with respect to logarithmic strength and load. The safe region is defined as the conditions when strength is larger than load, i.e. in log-scale

$$\ln(S) > \ln(L) . \quad (3)$$

We define the target function as the difference between the logarithmic strength and the load,

$$\ln(S) - \ln(L) , \quad (4)$$

a property that should exceed zero with a certain safety distance.

In the reference standard, the target is defined by means of required wall thickness  $e$ :

$$e = \frac{PS \cdot D_i}{2 \cdot f \cdot z - PS} + c_1 , \quad (5)$$

where  $PS$  is the *design pressure* in MPa,  $D_i$  is the inner diameter in mm,  $f$  is the material strength in MPa,  $z$  is a welding strength reduction factor, and  $c_1$  is the allowed corrosion in mm. By denoting the actual wall thickness by  $t = e - c_1$ , we can reformulate the requirement:

$$t > \frac{PS \cdot D_i}{2 \cdot f \cdot z - PS} \quad \text{to} \quad \frac{2 \cdot f \cdot z \cdot t}{D_i + t} > PS . \quad (6)$$

If we formulate this target by means of strength and load,  $\ln(S) > \ln(L)$ , we can define,

$$\ln(S) = \ln 2 + \ln f + \ln z + \ln t - \ln(D_i + t) \quad \text{and} \quad \ln(L) = \ln PS , \quad (7)$$

and obtain the target function,

$$\ln(S) - \ln(L) = \ln 2 + \ln f + \ln z + \ln t - \ln(D_i + t) - \ln PS . \quad (8)$$

Further, we can exclude the welding strength reduction factor by setting  $z = 1$ , since no welds are present in the actual case.

One could regard the *allowed corrosion*  $c_1$  to be a deterministic value to be accounted for in the investigation and change the nominal value of the wall thickness, 15.5 mm, according to this maximum corrosion to 14.5 mm. We will here instead regard the corrosion  $c_1$  as a random property, uniformly distributed between zero and one mm. The expected value of the corrosion reduction is then 0.5 mm and the nominal thickness becomes 15 mm.

The nominal values assumed in this report are summarized below. Note that they differ from actual values so as to avoid compromising CorPower's design.

$$\begin{aligned} z &= 1.0 & \ln z &= 0 \\ t &= 25 \text{ mm} & \ln t &= 3.22 \\ D_i &= 675 \text{ mm} & \ln D_i &= 6.51 \\ D_i + t &= 700 \text{ mm} & \ln(D_i + t) &= 6.55 \\ PS &= 12 \text{ MPa} & \ln PS &= 2.48 \end{aligned} \quad (9)$$

The nominal load, based on the design pressure is

$$\ln(L_{nom}) = \ln PS = \ln 12 = 2.48 . \quad (10)$$

The nominal material strength is found from the material specifications of S355J2H according to EN 10210-1:2006

Yield strength:  $R_{eH} = 345 \text{ MPa}$

Tensile strength:  $R_m = 490 \text{ MPa}$

There is no information in this specification about the uncertainty in strength values. To get such information we look for common practice in material strength specifications. We then find that the nominal values usually are given as minimum values. One example from the SSAB specifications for a similar steel in Table 1.

Table 1: Mechanical properties of SSAB S355J2H EN 10210.

Thickness (mm)	Yield Strength $R_{p0.2}$ (min MPa)	Tensile Strength $R_m$ (MPa)	Elongation A (min %)
2- 3	355	510- 680	22
3.01- 6.30	355	470- 630	22

The uncertainty is in some specifications of the tensile strength estimated to be contained in an interval and the typical width of such an interval is 20-30%. Assuming the specifications in our case are given as minimum values, we should change the nominal material strength according to this (using the lower percentage, 20%), giving

$$\begin{aligned} \overline{R_{eH}} &= \frac{345 \text{ MPa}}{0.9} = 383 \text{ MPa} & \overline{R_m} &= \frac{490 \text{ MPa}}{0.9} = 544 \text{ MPa} \\ \ln \overline{R_{eH}} &= 5.95 & \ln \overline{R_m} &= 6.30 \end{aligned} \quad (11)$$

This will result in nominal logarithmic values for the yield strength and for the tensile strength, respectively:

$$\begin{aligned} \ln(S_{nom,eH}) &= \ln 2 + \ln f + \ln z + \ln t - \ln(D_i + t) = 0.69 + 5.95 + 0 + 3.22 - 6.55 = 3.31 , \\ \ln(S_{nom,Rm}) &= \ln 2 + \ln f + \ln z + \ln t - \ln(D_i + t) = 0.69 + 6.30 + 0 + 3.22 - 6.55 = 3.66 . \end{aligned} \quad (12)$$

The yield strength result together with the nominal load are put in the VMEA spread sheet as shown in Table 2 for the yield stress case. These nominal values give the actual safety factors,

$$SF_{nom, eH} = \exp(3.31 - 2.48) = 2.3 \quad , \quad SF_{nom, Rm} = \exp(3.66 - 2.48) = 3.2 \quad . \quad (13)$$

Table 2: Nominal load and strength values for the yield strength case.

Input – Yield		Nominal safety factor	Nominal safety distance		
Median strength [MPa]	27.4		log strength	3.31	
Median load [MPa]	12.0		log load	2.48	
		Safety factor	2.28	Distance	0.82

## 2.2 Uncertainty sources identification

For the load we identify one uncertainty source:

- Valve relief pressure

For the strength (including tolerances) we identify five uncertainty sources:

- Material strength specification
- Inner diameter
- Initial wall thickness
- Amount of corrosion
- Model error due to thin-walled assumption

## 2.3 Uncertainty size and sensitivity assessment

### 2.3.1 Valve relief pressure

There may be a certain measurement error in the true valve relief value, which in the ordinary design should be covered by the *design pressure* of 125 bar, exceeding the nominal valve relief value with 5 bar. In the VMEA analysis we take this value as the basis for an uncertainty span, regarding relief pressure, of  $\pm 5$  bar, in relative terms giving the span

$$\frac{\pm 5 \text{ bar}}{120 \text{ bar}} = \pm 0.042 \quad (14)$$

and assuming a uniform distribution we obtain the uncertainty component

$$s_p = \frac{0.042}{\sqrt{3}} = 0.024 \quad . \quad (15)$$

Using the relative uncertainty corresponds to the uncertainty in the natural logarithm and the sensitivity is unity for both uncertainty sources above.

### 2.3.2 Material strength specification

In order to find the uncertainty of the actual material strength we use the considerations behind the estimated nominal material strength above. For the uncertainty we use the higher assessed percentage range of 30%. Assuming a uniform distribution, the standard deviation of the specified strengths can then be estimated to

$$s_{mat} = \frac{30\%}{2 \cdot \sqrt{3}} = 8.7\% , \quad (16)$$

where we use the relative uncertainty in accordance with the logarithmic definition of the target function, making the sensitivity coefficient to unity.

### 2.3.3 Inner diameter

The size is given by the tolerances which is assumed to be  $\pm 0.5$  mm. Modelling this as a uniform distribution gives the standard deviation

$$s_{D_i} = \frac{0.5 \text{ mm}}{\sqrt{3}} = 0.29 \text{ mm} . \quad (17)$$

The sensitivity with respect to the inner diameter is found by a difference quotient on the log-strength variable,  $\ln(S) = \ln 2 + \ln f + \ln z + \ln t - \ln(D_i + t)$

$$c_{D_i} = \left| \frac{-\ln(675+25) + \ln(675+25-2 \cdot 0.29)}{2 \cdot 0.29 \text{ mm}} \right| = 0.0014 \text{ mm}^{-1} , \quad (18)$$

where the other nominal values are constant and thus cancel out in the nominator difference.

### 2.3.4 Wall thickness

The wall thickness is another source of uncertainty, and it is influenced by both the geometric tolerance and corrosion.

We have no information about the geometric tolerances, and use the preliminary assumption from standard tolerances for hot rolled steel, see Table 3, taking the thickness tolerance to  $\pm 0.5$  mm, giving the uniform standard deviation

$$s_t = \frac{0.5 \text{ mm}}{\sqrt{3}} = 0.29 \text{ mm} . \quad (19)$$

The sensitivity with respect to the wall thickness is found by a difference quotient on the log strength,  $\ln(S) = \ln 2 + \ln f + \ln z + \ln t - \ln(D_i + t)$

$$c_t = \left| \frac{\ln 25 - \ln(675+25) - (\ln(25-2 \cdot 0.29) - \ln(675+25-2 \cdot 0.29))}{2 \cdot 0.29 \text{ mm}} \right| = 0.038 \text{ mm}^{-1} , \quad (20)$$

where constants cancel out in the nominator difference.

Table 3: Thickness tolerances according to EN 10051:2010, category B\*.

Nominal thickness $t$	Tolerances for a nominal width $w$			
	$w \leq 1\,200$	$1\,200 < w \leq 1\,500$	$1\,500 < w \leq 1\,800$	$w > 1\,800$
$t \leq 2,00$	$\pm 0,20$	$\pm 0,22$	$\pm 0,24$	–
$2,00 < t \leq 2,50$	$\pm 0,21$	$\pm 0,24$	$\pm 0,26$	$\pm 0,29$
$2,50 < t \leq 3,00$	$\pm 0,23$	$\pm 0,25$	$\pm 0,28$	$\pm 0,30$
$3,00 < t \leq 4,00$	$\pm 0,25$	$\pm 0,28$	$\pm 0,30$	$\pm 0,31$
$4,00 < t \leq 5,00$	$\pm 0,28$	$\pm 0,30$	$\pm 0,32$	$\pm 0,33$
$5,00 < t \leq 6,00$	$\pm 0,30$	$\pm 0,32$	$\pm 0,33$	$\pm 0,36$
$6,00 < t \leq 8,00$	$\pm 0,33$	$\pm 0,35$	$\pm 0,36$	$\pm 0,40$
$8,00 < t \leq 10,00$	$\pm 0,37$	$\pm 0,38$	$\pm 0,39$	$\pm 0,46$
$10,00 < t \leq 12,50$	$\pm 0,40$	$\pm 0,41$	$\pm 0,43$	$\pm 0,49$
$12,50 < t \leq 15,00$	$\pm 0,43$	$\pm 0,44$	$\pm 0,46$	$\pm 0,53$
$15,00 < t \leq 25,00$	$\pm 0,46$	$\pm 0,48$	$\pm 0,52$	$\pm 0,58$

\*Category B: specified minimum yield strength  $300\text{ MPa} < R_e \leq 360\text{ MPa}$ .

### 2.3.5 Amount of corrosion

We assume that the corrosion is uniformly distributed between zero and one mm, as discussed above, and obtain the standard deviation

$$s_{cor} = \frac{0.5\text{ mm}}{\sqrt{3}} = 0.29\text{ mm} . \quad (21)$$

The sensitivity coefficient for the log-strength with respect to corrosion is the same as for the geometric tolerance.

$$c_{cor} = c_t = 0.038\text{ mm}^{-1} \quad (22)$$

### 2.3.6 Model error due to thin-walled assumption

In addition, the reliability generating model may contain errors that need to be addressed. The structural model taken from the reference standard seems to be based on the thin-walled assumption regarding the hoop stress.

The model uncertainty introduced by the thin-wall assumption is judged to be  $\pm 2\%$  on the calculated stress, which assuming a uniform distribution results in the standard deviation

$$s_{model} = \frac{2\%}{\sqrt{3}} = 1.2\% = 0.012 . \quad (23)$$

### 2.3.7 Summary of uncertainty and sensitivity analysis

We put our findings in the VMEA spread sheet, see Table 4. For each source of uncertainty, the standard deviation and the sensitivity coefficient are multiplied resulting in an uncertainty with

respect to the target. In the spread sheet, the uncertainties are categorised as scatter (random variation) and other uncertainties, such as specification and model uncertainties.

Table 4: VMEA table of uncertainties.

Input					Result		
Uncertainty components	scatter	uncert.	Sensitivity coefficient c	standard deviation s	Scatter	Uncertainty	Total
<b>Strength</b>							
Material specification		x	1.000	0.087		0.087	
Diameter tolerance	x		0.001	0.289	0.000		
Thickness tolerance	x		0.038	0.231	0.009		
Thickness reduction corrosion	x		0.038	0.289	0.011		
Model error		x	1.000	0.012		0.012	
<b>Total Strength uncertainty</b>					<b>0.014</b>	<b>0.087</b>	<b>0.088</b>
<b>Load</b>							
Relief pressure		x	1.000	0.024		0.024	
<b>Total Load uncertainty</b>					<b>0.000</b>	<b>0.024</b>	<b>0.024</b>
<b>Total uncertainty</b>					<b>0.014</b>	<b>0.091</b>	<b>0.092</b>

## 2.4 Total uncertainty evaluation

The uncertainties are squared and added together to the overall statistical variance of the target. The square root of this variance is the statistical standard deviation, representing the overall uncertainty measure. In this case the total uncertainty is equal to 0.092, which is approximately 9% in terms of relative uncertainty, since we use natural logarithms.

## 2.5 Reliability and robustness evaluation

The statistical uncertainty measure is multiplied by the number 1.64 for the *statistical safety distance*. This is found as the “Variation distance 0.15” in the right columns in the sheet excerpt in Table 5, for the yield and UTS strength cases. If the nominal target function (the difference in logs, here 0.82 for yield strength) exceeds this number, then the design survival probability should be at least 95%.

Table 5: Reliability evaluation.

Input – Yield		Nominal safety factor		Nominal safety distance	
Median strength [MPa]	27.4			log strength	3.31
Median load [MPa]	12.0			log load	2.48
		Safety factor	2.28	Distance	0.82
Evaluation - Extra safely factor					
Reliability of 95%		Variation safety factor	1.16	Variation distance	0.15
Required extra safety factor		Extra safety factor	1.96	Extra distance	0.67
Input – UTS		Nominal safety factor		Nominal safety distance	
Median strength [MPa]	38.9			log strength	3.66
Median load [MPa]	12.0			log load	2.48
		Safety factor	3.24	Distance	1.17
Evaluation - Extra safely factor					
Reliability of 95%		Variation safety factor	1.16	Variation distance	0.15
Required extra safety factor		Extra safety factor	2.79	Extra distance	1.02

The amount of exceedance is a measure of the extra safety distance, here equal to  $0.82 - 0.15 = 0.67$  for the yield stress case, which should fulfil the designers demand about extra safety for approving the design. Figures 4 and 5 illustrate the safety distances. The corresponding safety factors (anti-logs) are given in the middle columns of Table 5. The nominal safety factors are 2.28 and 3.24 for the yield and ultimate strength limits respectively. The safety factor needed for the statistically based uncertainty is 1.16 in both cases which gives the extra safety factors 1.96 and 2.79, respectively.

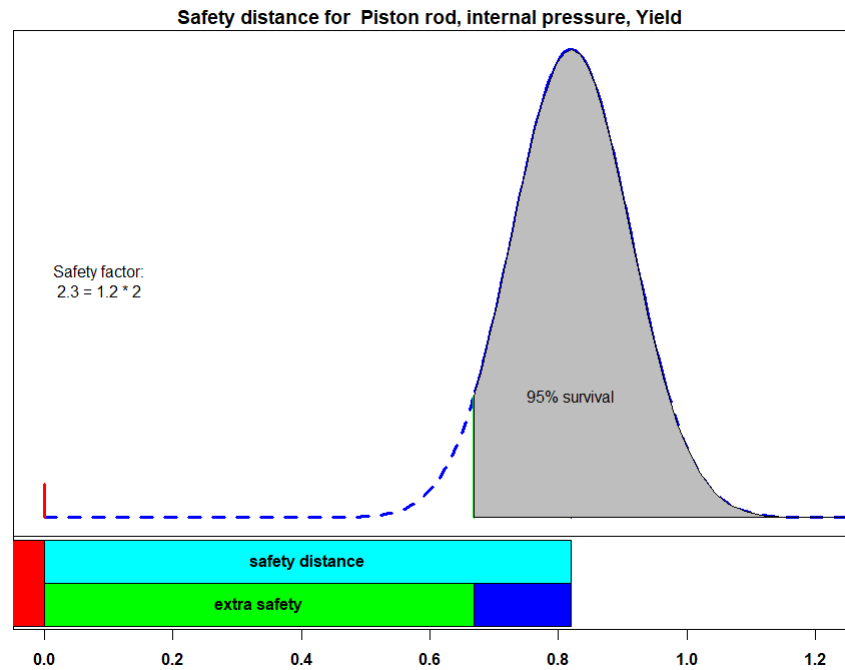


Figure 4: Safety distance for yield strength case.

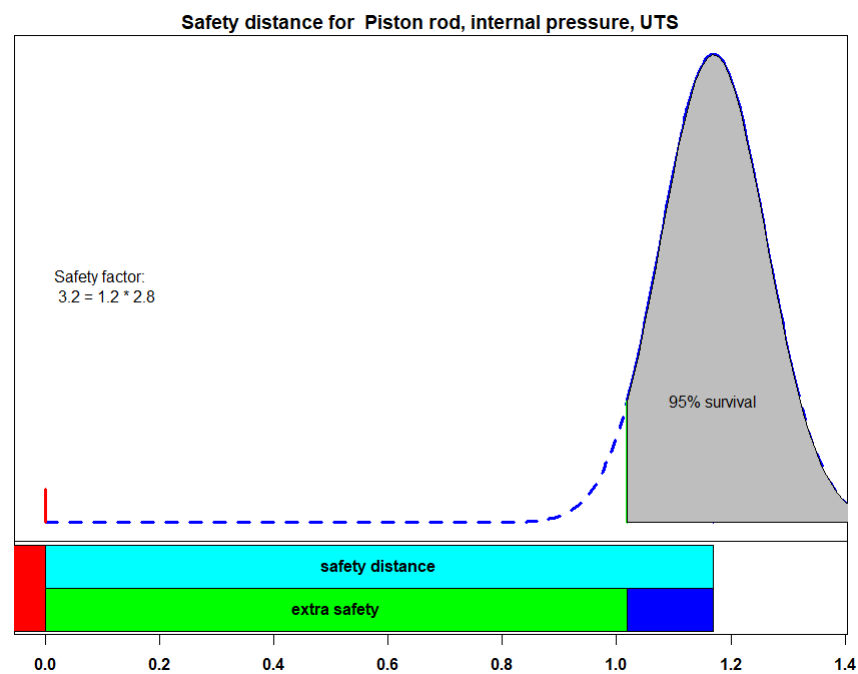


Figure 5: Safety distance for tensile strength case.

## 2.6 Conclusions and improvement actions

The large extra safety factors in this case would approve the design, which is in accordance with the result using the pressure vessel code. It is interesting to note that the safety factors used in the code are 1.5 and 2.4 on the yield and ultimate tensile strength, respectively. The code does not take geometric uncertainties into account nor model errors, which should make the VMEA safety margin lower. However, the calculations using the standard are based on minimal specified strength values which explain why the VMEA total margins actually are larger.

For possible improvements in terms of reducing uncertainty, it is helpful to compare the influence of the different uncertainty sources. In the uncertainty pie diagram in Figure 6, it can be seen that the uncertainty due to the specified material strength dominates the total uncertainty. The second largest uncertainty is due to valve relief pressure, and then the thickness reduction of corrosion. The tolerances and model errors have very small uncertainty influence.

As a conclusion, since the material strength specification dominates the total uncertainty, there is no need to put efforts on reducing the other uncertainties, especially the geometric tolerances are tight enough. Further, it is probably not worth the cost to require tighter material specifications, since the safety factors are good enough.

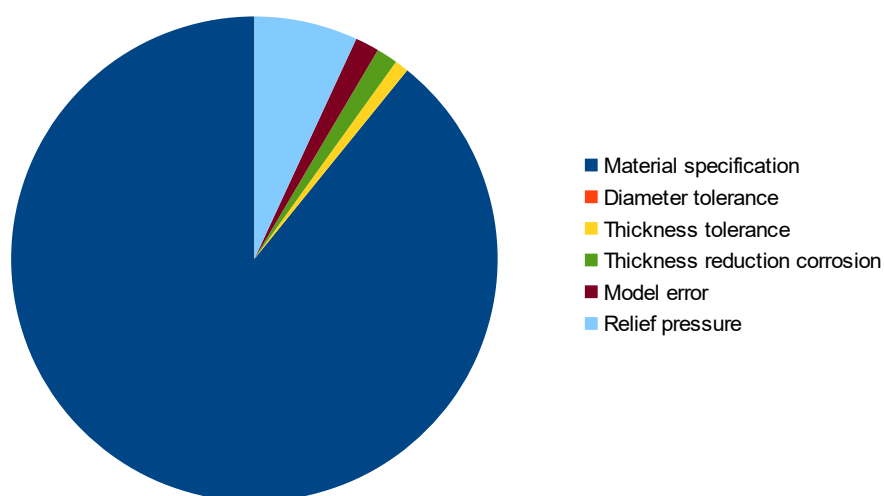


Figure 6: Pie chart of relative amount of uncertainty contributions.

### 3 Fatigue strength case

We will here re-evaluate the second case which represents fatigue strength design:

2. **Endurance limit calculation acc. to EN 13445-3:2014, Sect. 17. Calculation of the wall thickness of the pre-tension cylinder subjected to internal pressure.**

#### 3.1 Pressure vessel standard

Here, the failure mechanism to be considered is fatigue and the pressure vessel standard uses the Palmgren-Miner property “damage” as target. For the calculations they use a fatigue model including an asymptote at the “endurance” limit (ANL model). The fatigue life model for unwelded areas are used, which according to the standard is as follows:

$$N = \begin{cases} \left( \frac{46000}{\Delta \sigma_{fict} - 140} \right)^2 & \text{when } \Delta \sigma_{fict} > \Delta \sigma_D \\ 2 \cdot 10^6 \cdot \left( \frac{\Delta \sigma_D}{\Delta \sigma_{fict}} \right)^{10} & \text{when } \Delta \sigma_D \leq \Delta \sigma_{fict} \leq \Delta \sigma_{Cut} \\ \infty & \text{when } \Delta \sigma_{fict} < \Delta \sigma_{Cut} \end{cases} \quad (24)$$

where  $\Delta \sigma_{fict}$  is the so-called *fictive stress range* and the break point values are  $\Delta \sigma_D = 172.5$  MPa and  $\Delta \sigma_{Cut} = 116.7$  MPa. The three conditions above defines the *design curve*, the dashed blue line in Figure 7. The figure also illustrates some *validation data*, black dots, found in the comments to the standard. The fictive stress range is defined in the standard and calculated from the pressure, in the current case as

$$\Delta \sigma_{fict} = \frac{\Delta \sigma}{C_e \cdot C_T} \cdot K_f \quad (25)$$

where  $\Delta \sigma$  is the pseudo-elastic stress range calculated from the pressure cycle,  $C_e = 1$  is a correction factor for the wall thickness,  $C_T = 1$  is a correction factor for temperature, and  $K_f$  is the effective notch correction factor calculated as

$$K_f = 1 + \frac{1.5(K_t - 1)}{1 + 0.5 \cdot \max\left(1, K_t \frac{\Delta \sigma}{\Delta \sigma_D}\right)} \quad (26)$$

with the notch correction factor  $K_t = 1.4$ . Note that if  $\Delta \sigma \leq \Delta \sigma_D / K_t = 123.2$  MPa, then  $K_f = K_t = 1.4$ .

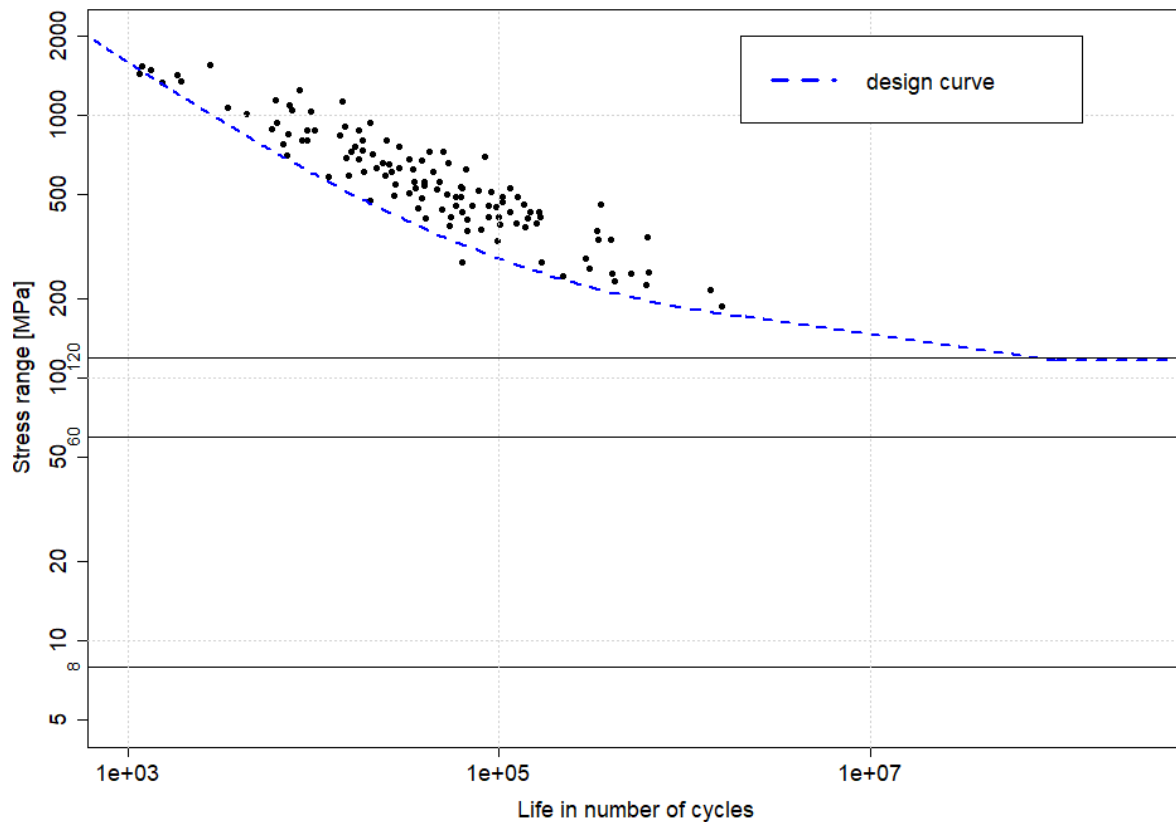


Figure 7: Design curve (dashed blue) and validation data (black dots).

### 3.1.1 Design loads

The *design loads* are derived by CorPower from three different operation cases of the equipment, and three load levels by means of the property *fictive stress range* are calculated. In this case study assumed loads are used in order not to reveal actual design loads. During the design life of one year it is assumed that the highest level occurs two hundred thousand times, the middle level five hundred thousand times, while the lowest level occurs two million times. This gives the load spectrum in Table 6, for one year of operation. In Figure 7, we have also added the three nominal load cycle ranges representing the design load cases, and it can be seen that only the largest one, 120 MPa, exceeds the defined fatigue limit. Note that the assumed load spectrum presented in Table 6 and Figure 7 differs from actual values so as to avoid compromising CorPower's design values.

Table 6: Design loads for one year.

Case, $i$	Fictive stress range, $\Delta \sigma_i$ [MPa]	Number of cycles, $n_i$
1	120	$2 \cdot 10^5$
2	60	$5 \cdot 10^5$
3	8	$2 \cdot 10^6$

### 3.1.2 Damage prediction

To assess the reliability, the damage according to the Palmgren-Miner rule is calculated. Only the largest load level contributes to the damage, which becomes

$$D = \frac{n_1}{N_1} = 2 \cdot 10^5 \cdot \frac{1}{2 \cdot 10^6} \cdot \left( \frac{120}{172.5} \right)^{10} = 0.00265 \quad (27)$$

corresponding to a predicted life of 377 years. It can be observed that the damage is far from unity, and thus the conclusion is that the design is well within the safe region.

However, by referring to fatigue “damage” the safety margin is related to life, which is very uncertain in the actual region, both because of lack of validation data and because of the vicinity to the highly uncertain “fatigue limit”. Since possible model errors are not taken into account in the methodology of the standard, the seemingly large safety margin may be misleading.

## 3.2 Target function definition and nominal values

For the VMEA reliability analysis we choose to consider the *equivalent strength and load* instead of the calculated damage and define the target function as

$$\ln S - \ln L \quad (28)$$

where  $S$  is the equivalent fatigue strength at two million cycles and  $L$  is the equivalent fatigue load range representing the target life. More precisely, the equivalent fatigue strength is the stress range corresponding to a life of two million cycles, and the equivalent fatigue load is the stress range (repeated two million times) that is damage equivalent to design loads corresponding to the target life. Note that the equivalent load and strength values are connected through the same number of cycle,  $n_0 = 2 \cdot 10^6$ .

### 3.2.1 Nominal strength

It is apparent that the *design curve* in Figure 7 represents a lower quantile of the fatigue strength distribution. For the VMEA analysis we need the median curve but the standard gives no information about which quantile that is used for the design curve. Thus, we need to find the median curve by fitting to the validation data by translating the design curve. A numerical search gives the best fit: multiplying the design life curve by a factor  $\gamma = 3.057$ , see the solid blue line in Figure 8. The standard deviation of the residuals in the curve fit is  $s = 0.59$ , and the design curve is then translated

$$\ln(\gamma)/s = \ln(3.057)/0.59 = 1.89 \quad (29)$$

standard deviations. Using a normal distribution assumption, it means that the design curve represents the 3% quantile in the distribution, i.e. 97% survival probability.

For the VMEA evaluations of reliability, the fatigue limit will be ignored by setting  $\Delta\sigma_{Cut}=0$  MPa , and thus the life model (best fit ANL model without fatigue limit) can be formulated as

$$N = \begin{cases} \gamma \cdot \left( \frac{46000}{\Delta\sigma_{fict} - 140} \right)^2 & \text{when } \Delta\sigma_{fict} > \Delta\sigma_D \\ \gamma \cdot n_0 \cdot \left( \frac{\Delta\sigma_D}{\Delta\sigma_{fict}} \right)^{10} & \text{when } \Delta\sigma_{fict} \leq \Delta\sigma_D \end{cases} \quad (30)$$

with  $n_0=2 \cdot 10^6$  . Using the fitted median curve above, we find the strength at two million cycles to failure by solving  $N=2 \cdot 10^6$  . Since we are in the region  $\Delta\sigma_{fict} > \Delta\sigma_D$  , the nominal equivalent fatigue strength becomes

$$S_{nom} = 46000 \cdot \sqrt{\frac{\gamma}{2 \cdot 10^6}} + 140 = 196.9 \text{ MPa} \quad (31)$$

which is illustrated in Figure 8.

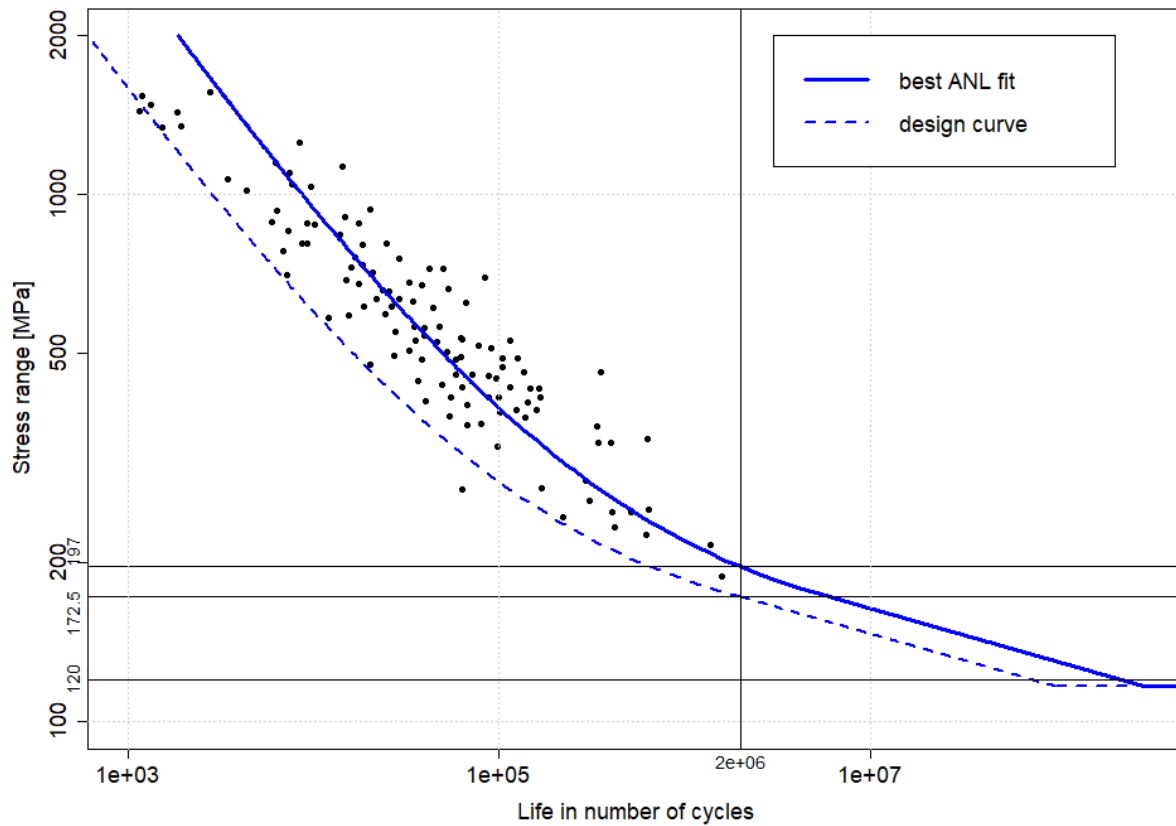


Figure 8: Median fatigue curves (solid), design curve (dashed) and validation data (dots).

### 3.2.2 Nominal load

We will here define the equivalent load that is represented by a stress range, repeated two million times, and is damage equivalent to design loads corresponding to the target life. The reliability investigation for the half-scale prototype is made with respect to a target design life of one year. The design loads in Table 6 will be translated into an equivalent fatigue load for the target life.

The damage for a target life of  $T$  years is denoted by  $D_T$  and is calculated using the ANL model without fatigue limit. For the case the equivalent fatigue load range,  $L_{eq}$  (at  $n_0=2 \cdot 10^6$  cycles), is below the break point of the ANL curve  $L_{eq} \leq \Delta \sigma_D$ , the damage due to the equivalent fatigue load can be found by

$$D_{eq} = n_0 \cdot \frac{1}{N(L_{eq})} = n_0 \cdot \frac{1}{\gamma n_0} \cdot \left( \frac{L_{eq}}{\Delta \sigma_D} \right)^{10} = \frac{1}{\gamma} \cdot \left( \frac{L_{eq}}{\Delta \sigma_D} \right)^{10} . \quad (32)$$

For a one-year design load spectrum  $\{(n_i, \Delta \sigma_i)\}_i$ , the  $T$ -year damage is calculated using Palmgren-Miner damage accumulation

$$D_T = T \cdot \sum_i \frac{n_i}{N_i(\Delta \sigma_i)} = T \cdot D_1 \quad (33)$$

where  $D_1$  is the damage for one year. Solving the damage equivalence equation,  $D_{eq} = D_T$ , gives

$$L_{eq,T} = (\gamma \cdot D_T)^{1/10} \cdot \Delta \sigma_D = (T \cdot \gamma \cdot D_1)^{1/10} \cdot \Delta \sigma_D . \quad (34)$$

For a design load spectrum with highest load level below  $\Delta \sigma_D = 172.5 \text{ MPa}$ , the one-year damage becomes

$$D_1 = \sum_i \frac{n_i}{N_i(\Delta \sigma_i)} = \sum_i n_i \cdot \frac{1}{\gamma \cdot n_0} \cdot \left( \frac{\Delta \sigma_i}{\Delta \sigma_D} \right)^{10} = \frac{1}{\gamma \cdot n_0 \cdot \Delta \sigma_D^{10}} \cdot \sum_i n_i \cdot \Delta \sigma_i^{10} = \frac{d}{\gamma \cdot n_0 \cdot \Delta \sigma_D^{10}} \quad (35)$$

where  $d$  is the so-called pseudo damage for one year and the  $T$ -year equivalent fatigue load becomes

$$L_{eq,T} = \left( \frac{T \cdot d}{n_0} \right)^{1/10} \quad \text{with} \quad d = \sum_i n_i \cdot \Delta \sigma_i^{10} . \quad (36)$$

As presented above in Table 6, the design loads are three load levels by means of the property *fictive stress range*. These refer to three different operation cases of the equipment. Since the highest load level  $\Delta \sigma_1 = 120 \text{ MPa}$  are below the break point  $\Delta \sigma_D = 172.5 \text{ MPa}$ , the one-year nominal equivalent fatigue load becomes

$$L_{nom} = L_{eq,1} = \left( \frac{d}{n_0} \right)^{1/10} = 95.3 \text{ MPa} . \quad (37)$$

### 3.2.3 Nominal safety factor

The nominal values for load and strength are put in the spreadsheet (Table 7), giving a nominal safety factor of 1.86. The next step is to investigate the requirement of the safety factor for design, which is based on the VMEA assessment.

Table 7: Nominal values for fatigue case.

Input		Nominal safety factor	
Design life [years]	1		
Median strength [MPa]	196.9		
Median load [MPa]	95.3		
		Safety factor	2.06

### 3.3 Uncertainty sources identification

#### 3.3.1 Strength uncertainties

We identify the following uncertainty sources for the strength

- Scatter
- Statistical uncertainty
- Relevance of validation data
- Model error in the fatigue model
- Wall thickness tolerances
- Wall thickness, corrosion

#### 3.3.2 Load uncertainties

The three load levels by means of *fictive stress range*, used as nominal load for strength comparison refer to three different operation cases of the equipment, the two with the highest levels occurring one million times each during the design life of one year.

The fictive stress range calculation includes two model assumptions that need to be considered, 1) the thin-walled assumption regarding hoop stress and 2) a chosen stress correction factor for notch influence.

Correction factors for temperature and wall thickness are regarded as negligible.

We consider the following uncertainties:

- Correspondence between the given pressures and the pressures generated by the three operations in service.
- Possible model error in the structural analysis.
- Possible error in the chosen notch correction factor.

### 3.4 Uncertainty size and sensitivity assessment

**Scatter and statistical uncertainty.** From the curve fit we found the scatter in terms of the standard deviation  $s=0.59$  for log-life. This number is based on 120 degrees of freedom and the t-correction can be neglected.

The statistical uncertainty by means of the calculated median curve is negligible with such a large sample.

The sensitivity coefficient with respect to the scatter is

$$c = \frac{\ln N^{-1}(2 \cdot 10^6) - (\ln N^{-1}(6.15 \cdot 10^5))}{2 \cdot 0.59} = \frac{\ln 197 - \ln 243}{2 \cdot 0.59} = -0.18 \quad , \quad (38)$$

Where  $6.15 \cdot 10^5$  is the life corresponding to a reduction of log target life by two standard deviations, see Figure 9.

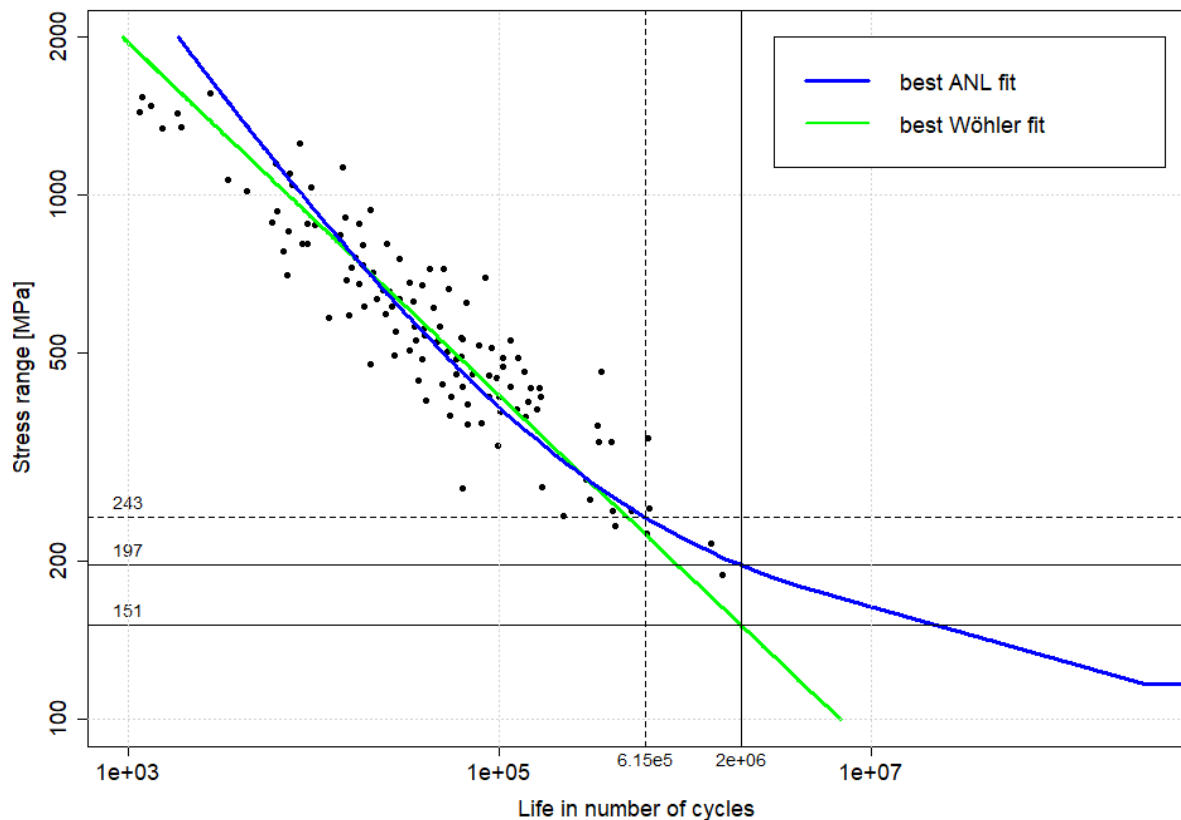


Figure 9: Wöhler fit (green), ANL fit (blue), design curve (dashed) and validation data (dots).

**Relevance of validation data.** The validation data appears to represent different steel qualities used in pressure vessels: “It may also be noted that the data base included vessels made from steels that ranged in tensile strength from 370 to 850 N/mm<sup>2</sup>.” The uncertainty regarding possible non-relevance to the actual material may then be assumed to be included in the large scatter.

**Model error in the fatigue model.** The model used here with an asymptote at the endurance limit is one of several models in fatigue practice. Other models include the *elementary* Wöhler model neglecting the fatigue limit, and *ordinary* Wöhler curve with other adjustments at the endurance limit. To find a reasonable estimate of the possible model error we here compare with the

*elementary* Wöhler curve without endurance limit considerations. Such a fitted curve is seen in Figure 9 as a green line.

We choose to evaluate the model uncertainty at two million cycles, where the fatigue strength of the ANL model is 197 MPa. It turns out that the elementary Wöhler fatigue strength at two million cycles is 151 MPa, and we regard these value as extreme estimates among models in the region of two million cycles. Assuming a model error uniformly distributed between the ANL model and the elementary Wöhler model gives the standard deviation

$$s_{model} = \frac{\ln(197) - \ln(151)}{\sqrt{12}} = 0.076 \quad (39)$$

**Wall thickness, tolerances and corrosion.** For the wall thickness we have the same uncertainties for fatigue strength as for the yield strength and ultimate strength, which were calculated above.

**Correspondence between the given pressures and the pressures generated by the three operations in service.** These pressures are assessed from numerical analyses and rig tests in laboratory. The real response from sea service is yet not known. We assign an uncertainty of  $\pm 5\%$  here and use the uniform distribution assumption to obtain the standard deviation

$$s_{service} = \frac{0.05}{\sqrt{3}} = 0.029 . \quad (40)$$

The sensitivity coefficient is unity in log scale, since fictive stress is proportional to pressure.

**Possible model error in the structural analysis.** The structural model taken from the reference standard seems to be based on the thin-walled assumption regarding the hoop stress. The model uncertainty introduced by this model is judged to be  $\pm 2\%$ , which assuming a uniform distribution results in the standard deviation

$$s_{hoop} = \frac{0.02}{\sqrt{3}} = 0.012 . \quad (41)$$

The sensitivity coefficient is also here unity in log scale.

**Possible error in the chosen notch correction factor.** The notch correction factor is chosen to

$$K_t = 1.4 . \quad (42)$$

This value is found in the pressure vessel standard:

At corners with small transition radii  $r$  (e.g. at base of forged/machined nozzles, see Figure 17-3), the following estimates of  $K_t$  may be assumed:

for  $r \geq e/4$ :

$$K_t = 1,4 \quad (17.6-7)$$

for  $r \geq e/8$ :

$$K_t = 1,8 \quad (17.6-8)$$

where

$e$  is the thickness of the thinner wall at the junction.

We don't know the radius of the sharpest junction in the piston rod. Since the next level in the standard is 1.8 we assume a maximum value in the middle of these two levels, 1.6. Using the uniform distribution we then assign the uncertainty

$$s_{K_i} = \frac{\ln(1.6) - \ln(1.4)}{\sqrt{3}} = 0.078, \quad (43)$$

with the sensitivity unity.

The uncertainty components are summarised in a spread sheet (Table 8) and a pie chart (Figure 10). The total uncertainty is estimated to 18% and is dominated by the strength scatter. Since it includes the uncertainty in the relevance of the used strength specification, which should be possible to reduce by getting relevant data. The other two important sources are possible model errors in the fatigue model ANL and in the stress concentration factor  $K_f$ . Both these model error uncertainties could probably also be reduced by further investigations.

Table 8: VMEA table for fatigue case.

Input					Result		
Uncertainty components	scatter	uncert.	Sensitivity coefficient c	standard deviation s	Scatter	Uncertainty	Total
<b>Strength</b>							
Strength scatter	x		0.177	0.590	0.104		
Strength model		x	1.000	0.076		0.076	
Wall thickness corrosion	x		0.038	0.289	0.011		
Wall thickness tolerance		x	0.038	0.289		0.011	
<b>Total Strength uncertainty</b>					<b>0.105</b>	<b>0.077</b>	<b>0.130</b>
<b>Load</b>							
Relevance to service		x	1.000	0.029		0.029	
Model error, structural		x	1.000	0.012		0.012	
Model error, Kt		x	1.000	0.078		0.078	
<b>Total Load uncertainty</b>					<b>0.000</b>	<b>0.084</b>	<b>0.084</b>
<b>Total uncertainty</b>					<b>0.105</b>	<b>0.114</b>	<b>0.155</b>

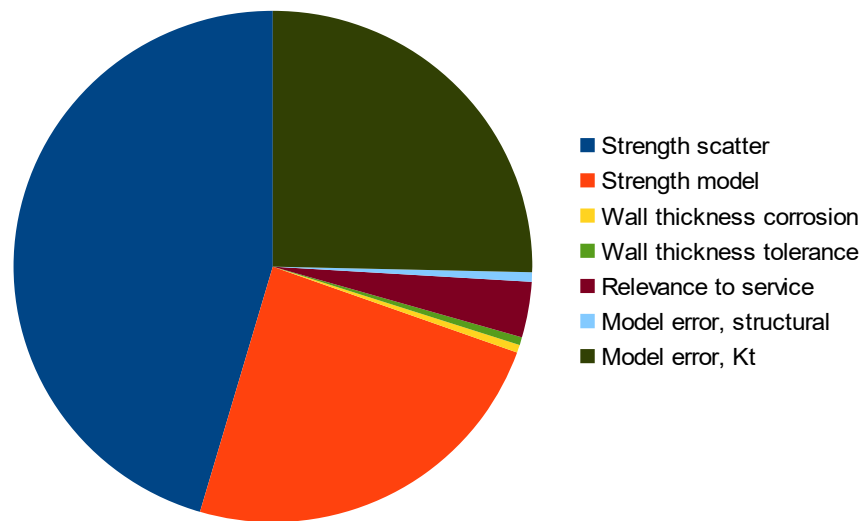


Figure 10: Pie chart of relative amount of uncertainty contributions for fatigue case.

### 3.5 Reliability and robustness evaluation

The nominal load and strength, together with the overall uncertainty gives a reliability assessment. The total uncertainty number 0.155 is multiplied by 1.64 to find the statistical safety distance corresponding to approximately 95% probability of survival,

$$0.155 \cdot 1.64 = 0.25 ,$$

which is given in the spread sheet below as “Variation distance”

Reducing the nominal safety distance with this number gives the extra safety distance,

$$0.73 - 0.25 = 0.47 .$$

The corresponding safety factor are the antilog of the distances and the total safety factor is 2.1. The safety factor needed for the statistical part is 1.3, which results in an extra safety factor of 1.6. This result can be regarded as safe, but the margin may not be as large as the damage evaluation by the standard suggests. The spread sheet (Table 9) summarises these results and it is also illustrated in Figure 11.

Table 9: Reliability evaluation for fatigue case.

Input		Nominal safety factor		Nominal safety distance	
Design life [years]	1			log strength	5.28
Median strength [MPa]	196.9			log load	4.56
Median load [MPa]	95.3				
		Safety factor	2.06	Distance	0.73
Evaluation - Extra safety factor					
Reliability of 95%		Variation safety factor	1.29	Variation distance	0.25
Required extra safety factor		Extra safety factor	1.60	Extra distance	0.47

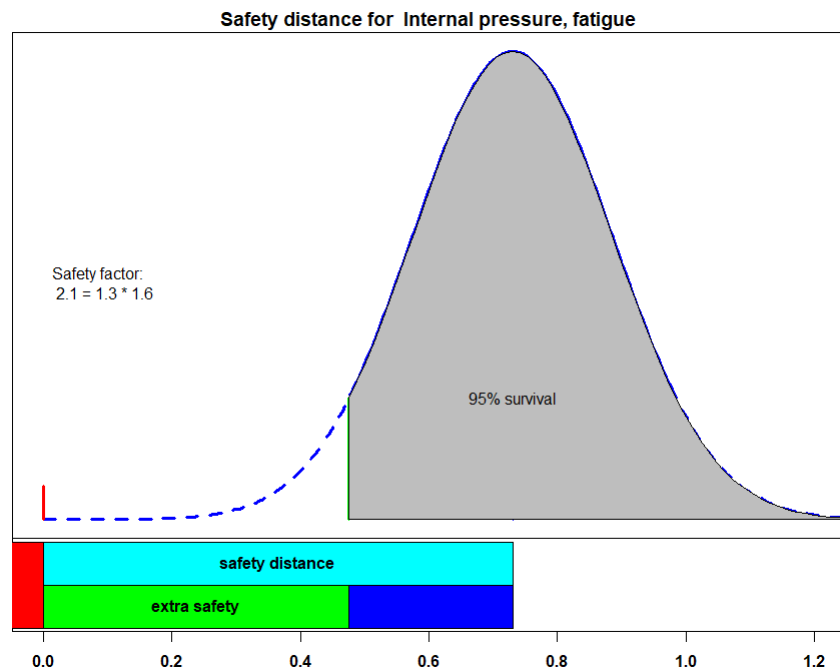


Figure 11: Reliability evaluation for the fatigue case.

### 3.5.1 Extension to twenty year life

It may also be interesting to extend the analysis to a future service situation. We choose to evaluate the reliability for twenty years. The nominal equivalent fatigue strength is defined as the load range at  $n_0 = 2 \cdot 10^6$  cycles and is thus unchanged. However, since the fatigue load accumulates over time, the equivalent fatigue load for 20 years will increase. Recall that the  $T$ -year equivalent fatigue load is calculated as

$$L_{eq,T} = \left( \frac{T \cdot d}{n_0} \right)^{1/10} \quad \text{with} \quad d = \sum_i n_i \Delta \sigma_i^{10} \quad (44)$$

which can be related to the one-year equivalent load as

$$L_{eq,T} = T^{1/10} \cdot L_{eq,1} \quad (45)$$

For  $T = 20$  years the nominal load becomes

$$L_{nom} = L_{eq,20} = 20^{1/10} \cdot L_{eq,1} = 1.35 \cdot 95.3 \text{ MPa} = 128.6 \text{ MPa} \quad (46)$$

which has increased by 35% compared to the one-year equivalent fatigue load.

The uncertainties in both load and strength do not change, since the uncertainties are modelled in log-scale. In the spread sheet in Table 10 the changes in the analysis are shown. The analysis shows that the extra safety factor is reduced to from 1.6 to 1.2. The result is also illustrated in Figure 12.

Table 10: Evaluation of twenty year reliability for the fatigue case.

Input		Nominal safety factor		Nominal safety distance	
Design life [years]	20				
Median strength [MPa]	196.9			log strength	5.28
Median load [MPa]	128.6			log load	4.86
		Safety factor	1.53	Distance	0.43

Evaluation - Extra safety factor				
Reliability of 95%	Variation safety factor	1.29	Variation distance	0.25
Required extra safety factor	Extra safety factor	1.19	Extra distance	0.17

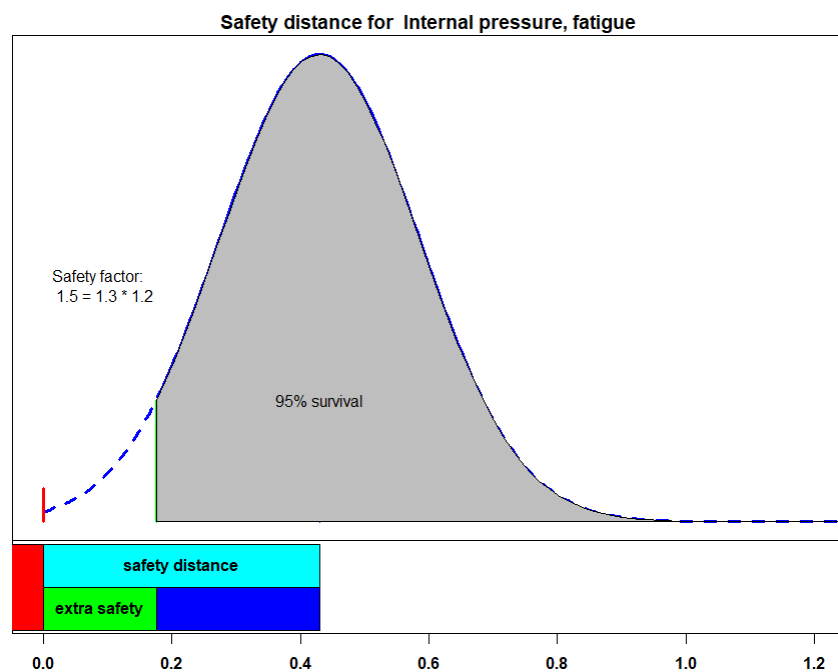


Figure 12: Twenty year reliability evaluation for the fatigue case.

## 4 Discussion

For the static strength example, the reliability methodology used by the pressure vessel standard is based on a standardised safety factor approach on the material strength. Since the material strength is specified as minimum values, more safety is added, but the definition of “minimum” seems not to be standardised and well defined. This gives problems when judging the amount of safety.

The VMEA evaluation aims to take all possible uncertainties into account. Lack of knowledge forces the analysis to use some approximations and judgements which weakens the conclusions. But, if needed, the largest uncertainty components can be identified and be subjected to further studies.

For the VMEA final result an extra safety factor is added, which can be difficult to interpret by means of amount of extra safety. In fact, the methodology is flexible, and demands regarding extra safety need to be discussed and specified for each specific application based on cost and safety requirements. Some guidance on choosing extra safety factors are found in (Johannesson, 2016).

For the fatigue strength example, the reliability methodology used by the pressure vessel standard is based on demands on the calculated damage. The margin given by this demand should include all scatter and uncertainty sources including possible model error. However, it may be doubtful if this standardised margin is valid for cases in the vicinity of the fatigue limit, since the possible model error here is substantial. The VMEA methodology, by choosing margins on the load dimension, gives a more robust judgement about model uncertainty and by comparing with another models its possible size can also be assessed.

The standard gives good guidance for performing design calculations, however it does not give any support on how new knowledge or data can be used in order to reduce the required safety margins or to guide improvement actions. On the contrary, for VMEA this kind of new information is directly reflected in the uncertainty numbers, and thus has a direct impact on the statistical safety margin. However, our recommendation is to complement the statistical safety factor by an extra safety factor not based on statistics but on economical and safety considerations. For both VMEA examples, the comparison of different uncertainty causes is a good basis for identifying possible new investigations or improvements actions that can give better knowledge on the uncertainties and hopefully result in a decrease of the overall uncertainty.

The VMEA methodology can also be a good basis for condition monitoring, where the accumulated equivalent fatigue load can be monitored and the uncertainties may be updated based on operational data. This gives the possibility to predict the remaining life and its uncertainty, which can be valuable input to the maintenance planning.

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# RiaSoR2



RELIABILITY IN A SEA OF RISK

## RiaSoR 2 project partners



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